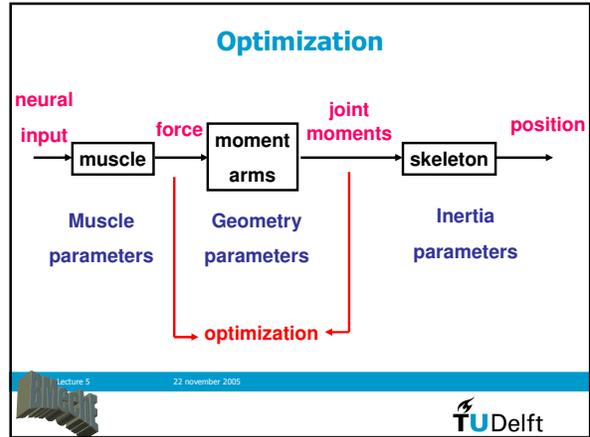


Human Motion Control

Lecture 5 Optimization

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Contents

- Muscle forces are calculated from joint moments through optimization
 - inverse dynamic optimization
 - forward dynamic optimization
- Optimization methods:
 - Local search
 - Global search
- Validation through EMG and NIRS



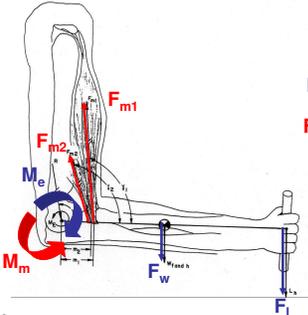
Why optimization?

- Optimization covers the (inverse dynamic) step from joint moments to muscle forces
- More muscles present than 'strictly' necessary
- many combinations of muscle force result in the same joint moment
- Motion equations:

$$\mathbf{A} \cdot \mathbf{f} = \mathbf{M}$$
 - A**: Moment arm matrix (m x n)
 - f**: muscle force vector (n x 1)
 - M**: Net moment vector (m x 1)
- Optimization: $n > m$
 - More unknowns than equations
 - Minimal number of muscles: $n_{\min} = m + 1$



Elbow model



$$M_e = F_w \cdot r_w + F_1 \cdot r_1$$

$$F_{m1} \cdot r_{m1} + F_{m2} \cdot r_{m2} = M_{en} = M_e$$

$$\mathbf{A} \cdot \mathbf{f} = \mathbf{M}$$


Optimization procedure

- Goal: Find one optimal solution for the combination of muscle forces
- Optimization: Minimization of criterion function, e.g. sum of squared muscle stresses

$$J = \sum (F_i / PCSA_i)^2$$
- Constraints:
 - $\mathbf{A} \cdot \mathbf{f} = \mathbf{M}$ Motion equations
 - $0 \leq f \leq f_{\max}$ Minimal and maximal force
 - $\mathbf{B}_{\text{lig}} \cdot \mathbf{f} > \mathbf{C}_1$ Linear inequalities (e.g. ligaments)
 - $\mathbf{Func}(\mathbf{f}) > \mathbf{C}_2$ Non-linear inequalities (e.g. joint reaction forces)



Optimization procedure

- Optimization criterion including constraints:

$$J = \Sigma (F_i/PCSA_i)^2 + \lambda_1^T \cdot (A \cdot f - M) + \lambda_2^T \cdot (B_{lig} \cdot f - C_1 + s) + \dots$$

λ_1, λ_2 : Lagrange multipliers
Good convergence (start small, ends large)

s : Slack variables

- $\partial J / \partial f = 0$ Minimal criterion value
- $\partial J / \partial \lambda = 0$ Equality constraints fulfilled
- $\partial J / \partial s = 0$ Inequality constraints fulfilled ($s > 0$)
- 1 equation per variable: System of equations can be solved!



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Optimization methods

- Local search
 - gradient methods
 - random search methods
- Global search
 - Grid search
 - Genetic algorithms

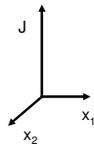


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Grid search



- Evaluate $J(x_1, x_2, \dots)$ for all values of x_1 and x_2
- Take smallest value of J : Global minimum
- Very, very elaborate search method



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Gradient methods

- Minimize $J = \text{function}(x)$ $\{x = F \text{ or } x = u\}$
- Optimum: $\frac{\partial J}{\partial x} = 0$

- Iterative procedure to proceed to optimum:
Steepest descent:

$$\vec{x}_{i+1} = \vec{x}_i - \alpha \cdot \frac{\partial J}{\partial \vec{x}}$$

- Newton algorithms:

$$\vec{x}_{i+1} = \vec{x}_i - \alpha \cdot H(\vec{x})^{-1} \cdot \frac{\partial J}{\partial \vec{x}}$$

$$H(\vec{x}) = \frac{\partial^2 J}{\partial \vec{x}_i \partial \vec{x}_j}$$

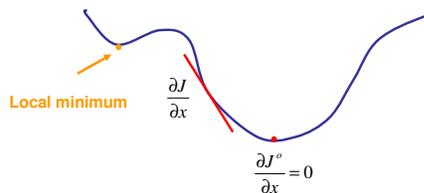


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Gradient methods



"Walking blindfolded downhill while feeling where the steepest slope is"
⇒ Risk of local minima



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Gradient methods

- Risk of local minima
- Gradients are calculated by numerical procedures:

$$\frac{\partial J}{\partial x} = \frac{J(x + \Delta x) - J(x)}{\Delta x}$$

- about 95% effort in calculating gradients
- Gradients have only local importance
- Most often used method



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Random search methods

- Random search direction v is chosen:

$$\vec{x}_{i+1} = \vec{x}_i - \alpha \cdot \vec{v}(x)$$

- New function evaluation:
if $J(x_{i+1}) < J(x_i)$ then take new value x_{i+1} in parameter space
 - No effort in calculating gradients
 - Progress depends on coincidence
 - Might be effective for large parameter systems



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Genetic algorithms

- Mimic nature in the search for the optimum: "Survival of the fittest"
- Start with a population of vectors x
 - Calculate criterion value $J(x)$
 - Select the best solutions of $J(x)$
 - Generate children (new population vectors x):
 - Combine vectors x : Cross-over of part of vector x (intermediate solutions: local)
 - Mutation of vector x (scatter solutions over workspace: global)



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Optimization criteria

- $J = J_{pc} + J_s$
- J_{pc} : Performance criteria (task-related)
 - Goal reached
 - Minimal time
 - maximal height
 - ...
- J_s : 'Load sharing' criteria (if task is submaximal)
 - Minimal energy consumption (difficult to calculate)
 - Energy related measures (easier to calculate)
 - sum of squared muscle forces
 - sum of squared muscle stresses
 - minimization of maximal muscle stress
 - Pain ?
 - ...



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Linear and non-linear criteria for load sharing

- Linear criteria
 - sum of muscle forces:
 $J = \Sigma F_i$
 - sum of muscle stresses σ :
 $J = \Sigma (F_i / PCSA_i)$
- Non-linear criteria
 - sum of squared muscle forces:
 $J = \Sigma (F_i)^2$
 - sum of muscle stresses σ :
 $J = \Sigma (F_i / PCSA_i)^2$
 - maximal endurance time T (e.g. related to $1/\sigma^3$)
 $J = \Sigma (F_i / PCSA_i)^3$



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Energy consumption (non-linear load sharing criterion)

- $E = A + M + S + W + D$
 - A is activation heat
 - M is maintenance heat
 - S is shortening heat
 - W is work done
 - D is energy dissipation in passive muscle tissue
- Hill-type muscle model is not very suited to predict muscle energy consumption

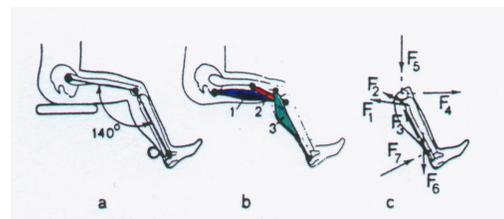


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Knee model (Dul et al., 1984)



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Knee model: Linear criteria (Dul et al., 1984)

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Linear criteria

- Solution: recruitment of minimum number of muscles
 - not realistic
 - if $J = \sum F_i$, then first the muscles with the largest moment arms are recruited.
 - If muscle force reaches maximum, the next muscle with the second largest moment arm is recruited, etc.
- Number of active muscles recruited:
 - \geq number of motion equations
 - \leq number of motion equations + number of inequality constraints

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Non-linear criteria

- Solution: synergism between muscles
- Force distribution depends on criterion:
 - many criteria result in comparable solution
 - mainly difference in magnitude of muscle force
 - whether or not active depends on favorable moment arm
- Feasible solution space is determined by equality and inequality constraints: Planes and half-planes.
- Comparison of criteria:
 - $\Sigma (F_i)^2 \Rightarrow$ Solution depends on number of muscle lines of action
 - $\Sigma (F_i/PCSA_i)^2 \Rightarrow$ Favors muscles with large PCSA and moment arm
 - Minimize σ_{max} : numerically unstable

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Conclusions (quasi-)static optimization

- Effect of muscle dynamics is neglected: Muscle force can increase instantaneously from zero to maximal
- Physical interpretation of optimization criteria is poor
- Linear optimization criteria result in muscle recruitment: Not realistic!
- Non-linear optimization criteria result in synergistic activity
- Number of non-linear criteria result in comparable solutions
- Muscle forces can not be validated!!

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Optimization

motions \rightarrow **body segments⁻¹** \rightarrow **Muscle⁻¹** \rightarrow neural input

forces

- Inverse static optimization (ISO)
- Inverse dynamic optimization (IDO)

neural input \rightarrow **Muscle** \rightarrow **body segments** \rightarrow motions

forces

- Forward static optimization (FSO)
- Forward dynamic optimization (FDO)

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Inverse & Forward dynamics optimization

- Inverse dynamic optimization (ISO & IDO):
 - **Algebraic** equations
 - (recorded) motion is given (comparable with task!)
 - Only **load sharing criteria** are used to solve for the redundant number of muscles
 - Should include inverse muscle dynamics
- Forward dynamic optimization (FSO & FDO):
 - **Differential** equations
 - Motion trajectory is calculated as a function of muscle forces
 - Performance criterion (task) is calculated as a result of the motion trajectory: Optimal motion trajectory
 - Both **performance criteria** and **load sharing criteria** are used to solve for the redundant number of muscles and redundant number of Degrees-of-Freedom

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Forward dynamic optimization

- Estimation of *optimal* neural input signal in time
 - k (time-frames) \times m (muscles) parameters
 - often parameterized into switch parameters for bang-bang control
- Results in *optimal* strategy and *optimal* kinematic pattern
- Allows for sensitivity analysis of model parameters: Effect of small parameter variations

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Optimization criterion

Forward dynamic optimization

- Minimize $J = [\text{taskvariables} + \text{time} + \text{energy}]$ as a function of neural input
 Calculate: $\frac{\partial J(t)}{\partial u_i(t)} = 0$ evaluation of differential equations: very time-consuming !!
- goal-directed: task variables are position errors, etc
- explosive: minimal time, maximal power, etc
- cyclic tasks or submaximal tasks: minimal energy
- Similar problem as the Central Nervous System is confronted with: Optimal motion is unknown !

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Inverse dynamics optimization including inverse muscle model

- Muscle force depends on previous muscle force
- Force can not increase or decrease with infinite speed
- Criterion $J = \Sigma (F_i / PCSA_i)^2$
- Constraints:
 - $A \cdot \mathbf{f} = \mathbf{M}$ motion equations
 - $\mathbf{f}_{\min} \leq \mathbf{f} \leq \mathbf{f}_{\max}$ minimal and maximal force depends on muscle dynamics

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Inverse dynamics including muscle dynamics

- Steps per time-interval:
 - Calculate \mathbf{f}_{\min} and \mathbf{f}_{\max} using *forward dynamic* muscle model with neural input $u = 0$ and $u = 1$, respectively.
 - Optimization: \mathbf{f} is calculated with \mathbf{f}_{\min} and \mathbf{f}_{\max} as constraints
 - Calculation of *actual* muscle states and neural input belonging to \mathbf{f} using *inverse dynamic* muscle model
 - Go to next time-step

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Inverse muscle model (IMM)

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Inverse muscle model

- Excitation:
 - Continuous form: $\dot{e} = \frac{u - e}{\tau_{ne}}$
 - Discrete form: $e_{i+1} = e_i + (u_i - e_i) \cdot (1 - e^{-\frac{dt}{\tau_{ne}}})$
- Activation:
 - Continuous form: $\dot{a} = \frac{u - a}{\tau_a}$
 - Discrete form: $a_{i+1} = a_i + (u_i - a_i) \cdot (1 - e^{-\frac{dt}{\tau_a}})$
- $\tau_a = \tau_{acr}$ if $e > a$ (increase of activation)
- $\tau_a = \tau_{dar}$ if $e < a$ (decrease of activation)

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Inverse muscle model

- Force-Velocity relation:

Relative Force :
$$F_r = \frac{F_{se}}{F_{max} \cdot act}$$

Inverse force - velocity relation : $v_{ce} = f(F_r)$

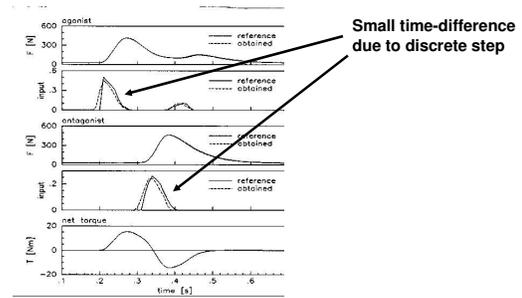
- Non-invertible relation: Find solution for *act* by iterative procedure



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Accuracy of IMM



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IFDO Inverse/Forward Dynamic Optimization

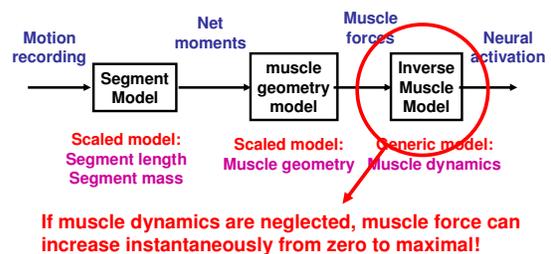
- Combine advantages of
 - Inverse dynamic \rightarrow Fast optimization
 - and
 - forward dynamic \rightarrow Better simulation



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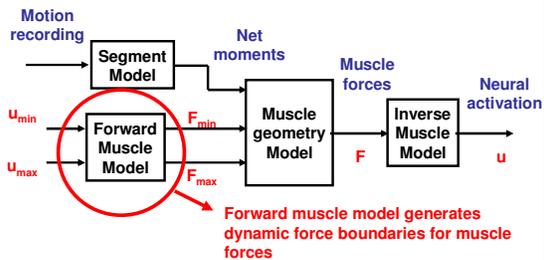
IFDO Inverse/Forward Dynamic Optimization including muscle dynamics



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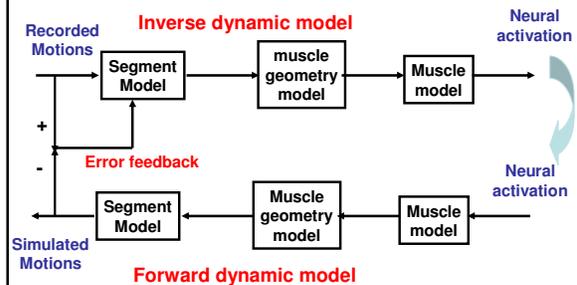
IFDO Inverse/Forward Dynamic Optimization including muscle dynamics



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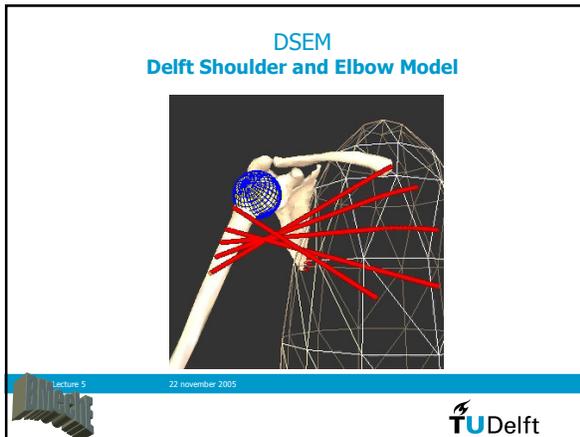
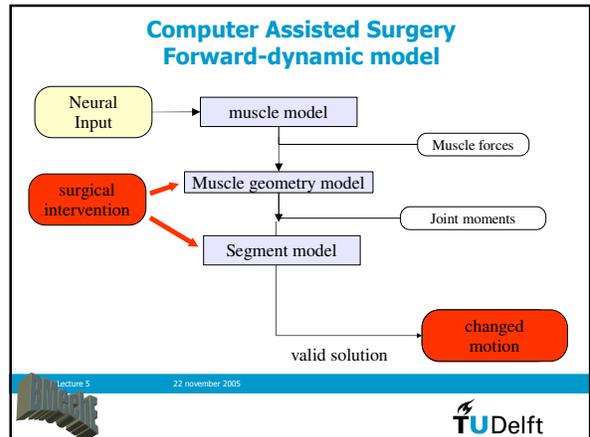
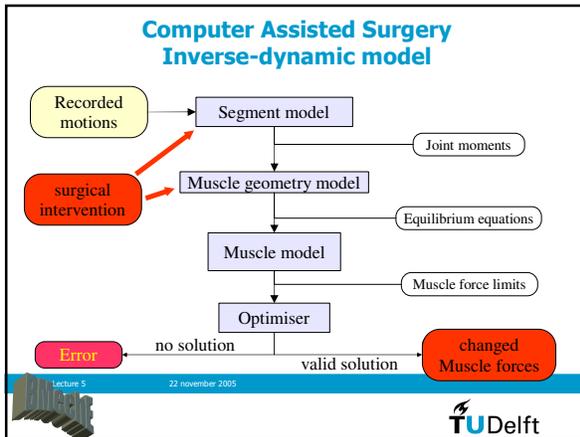
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IFDOC Inverse/Forward Dynamic Optimization with feedback error Correction

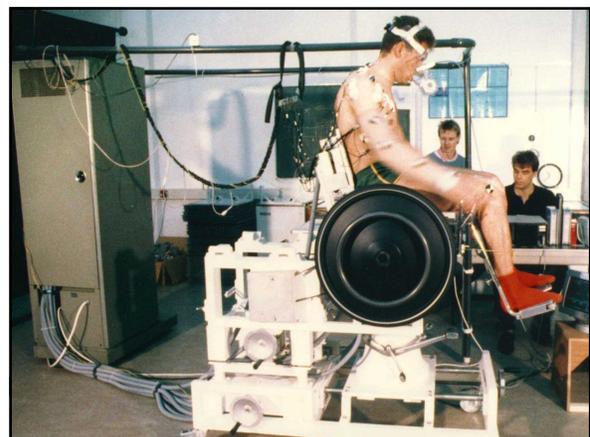
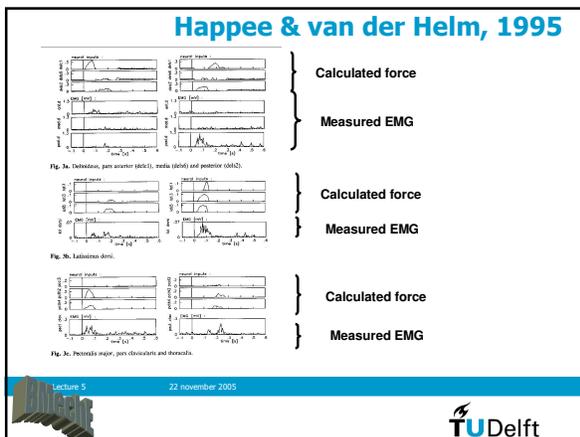


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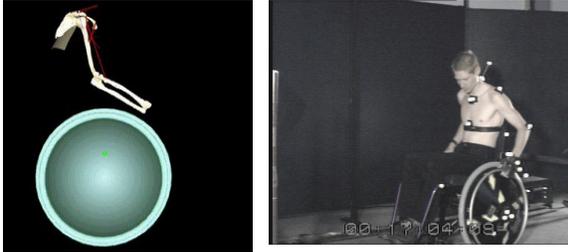
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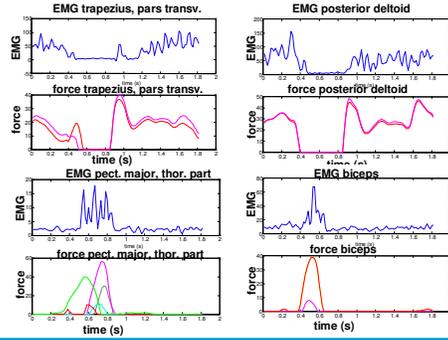
- ### Validation
- Muscle forces can not be measured: No strict validation!
 - EMG to force comparison:
 - Amplitude information not useful: Length & Velocity dependent
 - On-off patterns:
 - Wheelchair propulsion
 - Fast arm motions
 - Isometric, change in external force direction
 - Principal action for whole range of motion
 - Prediction of pathology and surgical outcome
 - Remaining function of paraplegics (Cleveland, OH)
 - Function after tendon transfer
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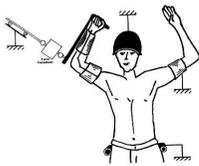
Output visualization: Wheelchair propulsion



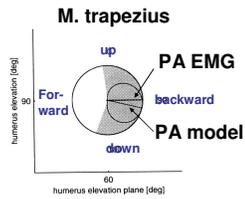
Wheelchair propulsion: EMG to force comparison



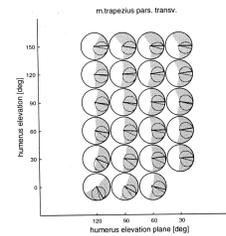
Principal action (PA): Preferred direction of muscle activation (Groot, 1998)



Experimental set-up



Principal action trapezius: whole range of motion (Groot, 1998)

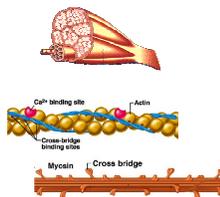


Optimization Criterion

1. Stress cost function

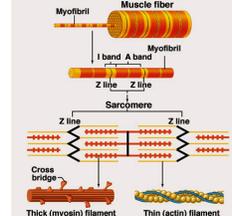
$$\text{MIN} \sum_{i=1}^n \left(\frac{F_i}{PCSA_i} \right)^2$$

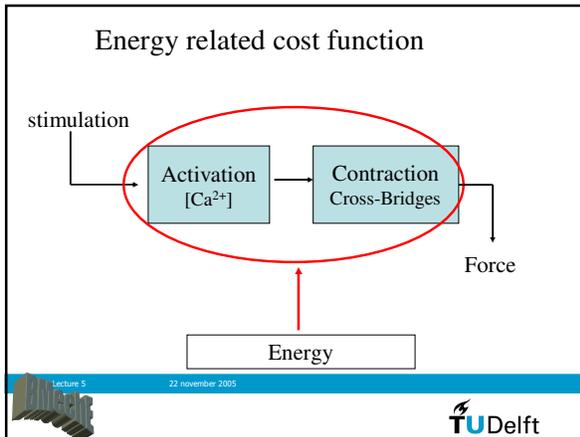
2. Nieuwe energie-gerelateerde costfunctie
 - Cross-bridge ontkoppeling
 - Terug pompen van Calcium



Muscle contraction

Filaments in a skeletal-muscle fiber





Energy related cost function

- Cross-Bridge detachment: $F \cdot v_{\text{zellenge}}$
 $= F \cdot V / PCSA = m \rho \cdot F / PCSA$
- Calcium uptake:
 $V \cdot \text{active state} \approx V \cdot (F / F_{\text{max}})^2 = m \rho \cdot (F / F_{\text{max}})^2$

$$\text{MIN} \sum_{i=1}^n m_i \left(c_1 \frac{F_i}{PCSA_i} + c_2 \left(\frac{F_i}{F_{i\text{max}}} \right)^2 \right)$$

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Optimization Criterion

- Stress cost function
- Energy related cost function

Near Infra Red Spectroscopy (NIRS)
 ↓
 Oxygen consumption

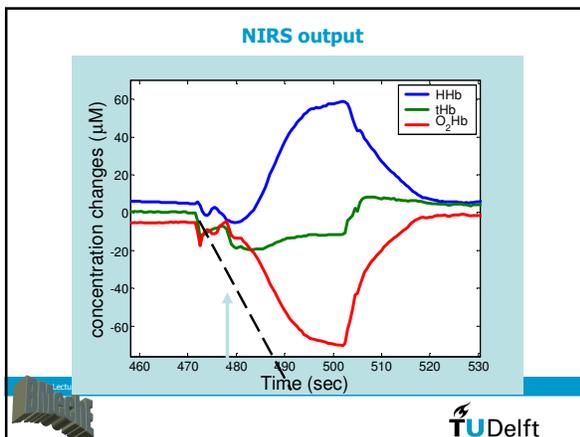
$$\text{MIN} \sum_{i=1}^n m_i \left(c_1 \frac{F_i}{PCSA_i} + c_2 \left(\frac{F_i}{F_{i\text{max}}} \right)^2 \right)$$

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Near Infra-Red Spectroscopy (NIRS)

- Non-invasive method
- tissue oxygenation
- oxygen dependent absorption
- [O₂Hb] and [HHb]

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Experimental Set-up

- 4 subjects
- 4 arm muscles BiB, BiL, BrR and TrL
- Isometric contractions: flexion/extension pro/supination
- 3 force levels: 10%, 25% and 50% MVC

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Set-up

- Position
- External forces and moments
- Force feedback
- Oxygen consumption (VO_2) per muscle (NIRS)
- (EMG)

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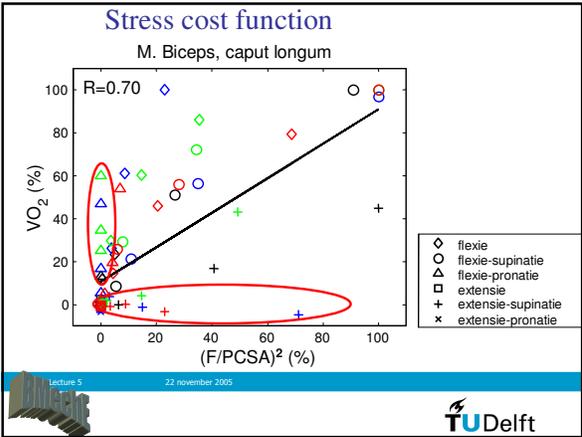
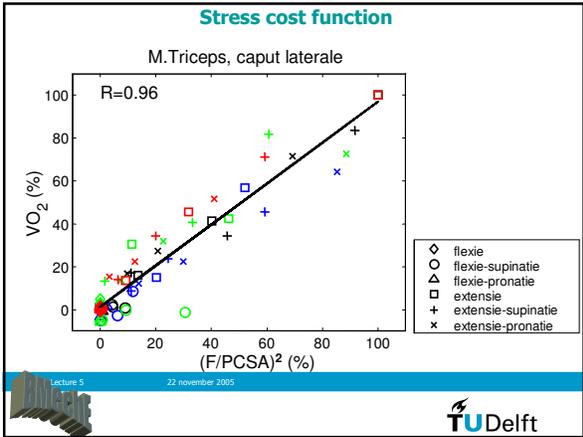
Model Simulations

Position → Dutch Shoulder-Elbow Model → Muscle forces 'Cost'

Forces Moments → Dutch Shoulder-Elbow Model

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Stress cost function

- Good agreement between VO_2 and cost for **m. triceps**
- Significant difference between VO_2 and cost for **flexors**
- pro/supination moments weigh too much in optimization

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